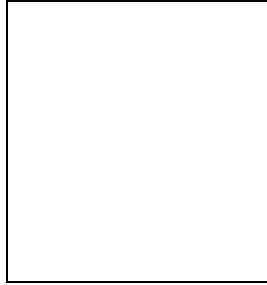


Multi gluon collinear limits from MHV amplitudes

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We consider the multi-collinear limit of multi-gluon QCD amplitudes at tree level. We use the MHV rules for constructing colour ordered tree amplitudes and the general collinear factorisation formula to derive timelike splitting functions that are valid for specific numbers of negative helicity gluons and an arbitrary number of positive helicity gluons (or vice versa).

1 Introduction

This talk is based on our recent paper [5], where we exploit the MHV formalism to examine the singularity structure of tree-level amplitudes when many gluons are simultaneously collinear.

The interpretation of $\mathcal{N} = 4$ supersymmetric Yang-Mills theory and QCD as a topological string propagating in twistor space [1], has inspired a new and powerful framework for computing tree-level and one-loop scattering amplitudes in Yang-Mills gauge theory. Notably, two distinct formalisms have been developed for calculations of scattering amplitudes in gauge theory – the ‘MHV rules’ of Cachazo, Svrček and Witten (CSW) [2], and the ‘BCF recursion relations’ of Britto, Cachazo, Feng and Witten [3, 4].

Understanding the infrared singular behaviour of multi-parton amplitudes is a prerequisite for computing infrared-finite cross sections at fixed order in perturbation theory. In general, when one or more final state particles are either soft or collinear, the amplitudes factorise. The first factor in this product is an amplitude depending on the remaining hard partons in the process (including any hard partons constructed from an ensemble of unresolved partons). The second factor contains all of the singularities due to the unresolved particles. One of the best known examples of this type of factorisation is the limit of tree amplitudes when two particles are collinear. This factorisation is universal and can be generalised to any number of loops [6].

A useful feature of the MHV rules is that it is not required to set reference spinors η_α and $\eta_{\dot{\alpha}}$ to specific values dictated by kinematics or other reasons. In this way, on-shell (gauge-

invariant) amplitudes are derived for arbitrary η 's, i.e. without fixing the gauge. By starting from the appropriate colour ordered amplitude and taking the collinear limit, the full amplitude factorises into an MHV vertex multiplied by a multi-collinear splitting function that depends on the helicities of the collinear gluons. Because the MHV vertex is a single factor, the collinear splitting functions have a similar structure to MHV amplitudes. Furthermore, the gauge or η -dependence of the splitting function drops out.

One of the main points of our approach is that, in order to derive all required splitting functions we do not need to know the full amplitude. Out of the full set of MHV-diagrams contributing to the full amplitude, only a subset will contribute to the multi-collinear limit. This subset includes only those MHV-diagrams which contain an internal propagator which goes on-shell in the multi-collinear limit. In other words, the IR singularities in the MHV approach arise entirely from internal propagators going on-shell. This observation is specific to the MHV rules method and does not apply to the BCF recursive approach.

The basic building blocks of the MHV rules approach [2] are the colour-ordered n -point vertices which are connected by scalar propagators. These MHV vertices are off-shell continuations of the maximally helicity-violating (MHV) n -gluon scattering amplitudes of Parke and Taylor [7, 8]. They contain precisely two negative helicity gluons. Written in terms of spinor inner products, they are composed entirely of the holomorphic products $\langle i j \rangle$ of the right-handed (undotted) spinors, rather than their anti-holomorphic partners $[i j]$,

$$A_n(1^+, \dots, p^-, \dots, q^-, \dots, n^+) = \frac{\langle p q \rangle^4}{\langle 1 2 \rangle \langle 2 3 \rangle \dots \langle n-1, n \rangle \langle n 1 \rangle}, \quad (1)$$

where we introduce the common notation $\langle p_i p_j \rangle = \langle i j \rangle$ and $[p_i p_j] = [i j]$. By connecting MHV vertices, amplitudes involving more negative helicity gluons can be built up.

The factorisation properties of amplitudes in the infrared play several roles in developing higher order perturbative predictions for observable quantities. First, a detailed knowledge of the structure of unresolved emission enables phase space integrations to be organised such that the infrared singularities due to soft or collinear emission can be analytically extracted. Second, they enable large logarithmic corrections to be identified and resummed. Third, the collinear limit plays a crucial role in the unitarity-based method for loop calculations. In general, to compute a cross section at N^n LO, one requires detailed knowledge of the infrared factorisation functions describing the unresolved configurations for n -particles at tree-level, $(n-1)$ -particles at one-loop etc.

2 Examples

In this section I'll give some simple examples of how to obtain the splitting amplitudes starting from the general factorisation formula. For more details and the full results for up to six gluons please see ref. [5]. The general factorisation formula describes the behaviour of the amplitude in the collinear limit. When n particles become collinear the amplitude is given by

$$A_N(1^{\lambda_1}, \dots, N^{\lambda_N}) \rightarrow \text{split}(1^{\lambda_1}, \dots, n^{\lambda_n} \rightarrow P^\lambda) \times A_{N-n+1}((n+1)^{\lambda_{n+1}}, \dots, N^{\lambda_N}, P^\lambda).$$

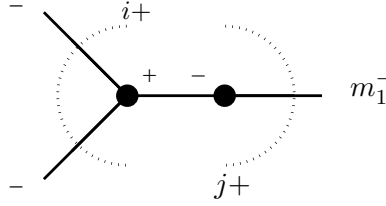
where split denotes the splitting function. The simplest example is the case of $\text{split}(1^+, \dots, n^+ \rightarrow P^+)$ which can be obtained directly from a single MHV vertex namely

$$A(1^+, \dots, n^+, (n+1)^+, (n+2)^-, (n+3)^-) = \frac{\langle n+2, n+3 \rangle^4}{\prod_{i=1}^{n+2} \langle i i+1 \rangle}. \quad (2)$$

Now taking particles $1, \dots, n$ collinear, assuming that

$$\sum_{i=1}^n p_i = P \quad \text{and} \quad p_i \rightarrow z_i P \quad \Rightarrow \quad \langle i n+1 \rangle \rightarrow \sqrt{z_i} \langle P n+1 \rangle \quad (3)$$

Figure 1: NMHV diagrams contributing to $\text{split}(1^+, \dots, i^-, \dots, n^+ \rightarrow P^-)$. Negative helicity gluons are indicated by solid lines, while arbitrary numbers of positive helicity gluons emitted from each vertex are shown as dotted arcs.



in the collinear limit, we obtain

$$A_{n+3}(1^+, \dots, n^+, (n+1)^+, (n+2)^-, (n+3)^-) = \frac{\langle n+2 \ n+3 \rangle^4}{\underbrace{\langle P \ n+1 \rangle \langle n+1 \ n+2 \rangle \langle n+2 \ n+3 \rangle \langle n+3 \ P \rangle}_{A_4((n+1)^+, (n+2)^-, (n+3)^-, P^+)}} \times \frac{1}{\underbrace{\sqrt{z_1} \sqrt{z_n} \prod_{i=1}^{n-1} \langle i \ i+1 \rangle}_{\text{split}(1^+, \dots, n^+ \rightarrow P^+)}}$$

where we can directly read off the splitting function. Similarly we can obtain the splitting function $\text{split}(1^+, \dots, i^-, \dots, n^+ \rightarrow P^-)$.

$$\text{split}(1^+, \dots, i^-, \dots, n^+ \rightarrow P^-) = \frac{z_i^2}{\sqrt{z_1} \sqrt{z_n} \prod_{i=1}^{n-1} \langle i \ i+1 \rangle} \quad (4)$$

by starting from the amplitude $A(1^+, \dots, i^-, \dots, n^+, (n+1)^+, (n+2)^-, (n+3)^-)$.

A more complicated example is the splitting function $\text{split}(1^+, \dots, i^-, \dots, n^+ \rightarrow P^+)$. This splitting amplitude can not be obtained from MHV amplitudes alone. In this case we have to consider NMHV amplitudes which yields to more complicated expressions. In general the difficulty of the problem increases with ΔM , which is given by the change of the number of gluons with negative helicities. The first two examples were of the type $\Delta M = 0$, while this example is of the type $\Delta M = 1$. In this case only the diagram in figure 1 contributes since it is the only diagram in which the propagator becomes onshell in the collinear limit. Starting with the MHV representation of this diagram and performing the same replacements as above we obtain

$$\text{split}(1^+, \dots, i^-, \dots, n^+ \rightarrow P^+) = \frac{1}{\sqrt{z_1} \sqrt{z_n} \prod_{l=1}^{n-1} \langle l, l+1 \rangle} \left(\sum_{i=0}^{m_1-1} \sum_{j=m_1}^n \frac{\Delta_{(1)}(i, j; m_1)^4}{D(i, j, q_{i+1, j})} \right), \quad (5)$$

where we define

$$D(i, j, q) = \frac{q_{i+1, j}^2}{\langle i, i+1 \rangle \langle j, j+1 \rangle} \Delta_{(1)}(i, j; i) \Delta_{(1)}(i, j; i+1) \Delta_{(1)}(i, j; j) \Delta_{(1)}(i, j; j+1). \quad (6)$$

and

$$\Delta_{(1)}(i, j; a) = \sum_{l=i+1}^j \langle a \ l \rangle \sqrt{z_l}. \quad (7)$$

Specific results can immediately be obtained from this expression, e.g.

$$\begin{aligned} \text{split}(1^-, 2^+, 3^+ \rightarrow P^+) &= \frac{\langle 1 \ 2 \rangle z_2^2}{\sqrt{z_1} \sqrt{z_2} \sqrt{z_3} s_{1,2} (z_1 + z_2) (\langle 1 \ 3 \rangle \sqrt{z_1} + \langle 2 \ 3 \rangle \sqrt{z_2})} \\ &+ \frac{(\langle 1 \ 2 \rangle \sqrt{z_2} + \langle 1 \ 3 \rangle \sqrt{z_3})^3}{s_{1,3} \langle 1 \ 2 \rangle \langle 2 \ 3 \rangle (\langle 1 \ 3 \rangle \sqrt{z_1} + \langle 2 \ 3 \rangle \sqrt{z_2})} \end{aligned} \quad (8)$$

3 Conclusion

In [5] we have considered the collinear limit of multi-gluon QCD amplitudes at tree level. We have used the new MHV rules for constructing colour ordered amplitudes from MHV vertices together with the general collinear factorisation formula to derive timelike splitting functions that are valid for specific numbers of negative helicity gluons with an arbitrary number of positive helicity gluons (or vice versa). In this limit, the full amplitude factorises into an MHV vertex multiplied by a multi-collinear splitting function that depends on the helicities of the collinear gluons. These splitting functions are derived directly using MHV rules. Out of the full set of MHV-diagrams contributing to the full amplitude, only the subset of MHV-diagrams which contain an internal propagator which goes on-shell in the multi-collinear limit contribute.

We find that the splitting functions can be characterised by ΔM , the difference between the number of negative helicity gluons before taking the collinear limit, and the number after. $\Delta M + 1$ also coincides with the number of MHV vertices involved in the splitting functions. Our main results are splitting functions for arbitrary numbers of gluons where $\Delta M = 0, 1, 2$. Splitting functions where the difference in the number of positive helicity gluons $\Delta P = 0, 1, 2$ are obtained by the parity transformation. These general results are sufficient to describe *all* collinear limits with up to six gluons. We have given explicit results for up to four collinear gluons for all independent helicity combinations, which numerically agree with the results of Ref. [9], together with new results for five and six collinear gluons. This method can be applied to higher numbers of negative helicity gluons, and via the MHV-rules for quark vertices, to the collinear limits of quarks and gluons [10].

Acknowledgments

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